

Inferring regulatory networks using multiple data sources.

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Can we reconstruct the network from the data?

Can we improve the reconstructed network by using additional sources of information as prior biological knowledge?

What is a Regulatory network?



•Set of nodes that regulate each other.

•Edges represent putative causal relationships.

•Only measured elements are represented as nodes.

•Intermediary elements that are not measured are not represented.



















Find the best structure $\mathcal{M}^* = \operatorname{argmax}_{\mathcal{M}} \{ \mathcal{P}(\mathcal{M}|\mathcal{D}) \}$

Find the best parameters $q = \operatorname{argmax}_{q} \{ P(q | \mathcal{M}^*, \mathcal{D}) \}$

 $P(\mathcal{M}|\mathcal{D}) \propto P(\mathcal{D}|\mathcal{M})P(\mathcal{M})$

 $P(\mathcal{D}|\mathcal{M}) = \int P(\mathcal{D}|\mathbf{q}, \mathcal{M}) P(\mathbf{q}|\mathcal{M}) d\mathbf{q}$ BGe - Bayesian Gaussian equivalence scores BDe - Bayesian Discretized equivalence scores

BioSS



BioSS

One approach would be calculate $P(\mathcal{M}|\mathcal{D})$ to all possible structures \mathcal{M}^*

Problems:

Number of nodes	2	4	6	8	10
Number of topologies	3	543	3.7×10^{6}	7.8×10^{11}	4.2×10^{18}

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Bayesian networks + MCMC



•Marriage between graph theory and probability theory.

BioSS

•It is possible to score a network in light of data. We can assert how well a particular network explains some observed data.

•We use Markov Chain Monte Carlo (MCMC) for sampling networks.

•There are problems with equivalence classes...

Bayesian networks Equivalence classes

В



•Score of first three networks are the same.

•They can't be distinguished in light of the data.

•We can only learn the undirected graph.

 Unless... we use interventions or prior knowledge.





How can biological prior knowledge be integrated in the Bayesian networks?





MCMC and Priors

$$A = \min\left\{\frac{P(D|G')}{P(D|G)}\frac{P(G')}{P(G)}\frac{Q(G|G')}{Q(G'|G)}, 1\right\}$$

BioSS

We model the prior with the Gibbs distribution:

$$P(G|\beta) = \frac{e^{-\beta E(G)}}{Z(\beta)}$$

Where the partition function is:

$$Z(\beta) = \sum_{G \in \mathcal{G}} e^{-\beta E(G)}$$

Imoto,S., Higuchi,T., Goto,T., Kuhara,S. and Miyano,S. (2003) Combining microarrays and biological knowledge for estimating gene networks via Bayesian networks. *Proceedings IEEE Computer Society Bioinformatics* Conference, (CSB'03), 104–113.

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Hyperparameter

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The energy

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Biological Prior Knowledge

Biological prior knowledge matrix

$$B=egin{pmatrix} b_{11}&b_{12}&b_{13}&\cdots &b_{1n}\ b_{21}&b_{22}&b_{23}&\cdots &b_{2n}\ b_{31}&b_{32}&b_{33}&\cdots &b_{3n}\ dots&dots$$

$$0 \le b_{ij} \le 1$$

 b_{ij} Indicates some knowledge about the relationship between genes *i* and *j*



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 b_{ij} Indicates some knowledge about the relationship between genes *i* and *j*

 $E(G) = \sum_{i,j=1}^{N} |B_{i,j} - G_{i,j}|$

BioSS

Define the energy of a Graph G

$$G = \begin{pmatrix} g_{11} & g_{12} & g_{13} & \cdots & g_{1n} \\ g_{21} & g_{22} & g_{23} & \cdots & g_{2n} \\ g_{31} & g_{32} & g_{33} & \cdots & g_{3n} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ g_{n1} & g_{n2} & g_{n3} & \cdots & g_{nn} \end{pmatrix} \overset{g_{ij} \in \{0, 1\}}{E(G)}$$

Sample graph and the hyperparameter β .

$$A = \min\left\{\frac{P(D|G')P(G'|\beta')P(\beta')}{P(D|G)P(G|\beta)P(\beta)}, 1\right\}$$

- 1. Sample graph with β fixed.
- 2. Sample β with graph fixed.

$$A_{1} = \min\left\{\frac{P(D|G')P(G'|\beta)}{P(D|G)P(G|\beta)}, 1\right\}$$
$$A_{2} = \min\left\{\frac{P(G|\beta')P(\beta')}{P(G|\beta)P(\beta)}, 1\right\}$$

$$A_2 = \min\left\{\frac{e^{-\beta' E(G)}}{Z(\beta')} \frac{Z(\beta)}{e^{-\beta E(G)}}, 1\right\} = \min\left\{e^{-E(G)(\beta'-\beta)} \frac{Z(\beta)}{Z(\beta')}, 1\right\}$$



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•How to calculate the partition function? How to sum over BioSS all possible graphs?

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Rewrite the energy as a function of nodes and parent sets

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Rewrite the energy as a function of nodes and parent sets

$$E(G) = \sum_{n=1}^{N} \mathcal{E}(n, \pi_n[G])$$

$$Z = \sum_{G \in \mathcal{G}} e^{-\beta E(G)}$$

$$\approx \sum_{\pi_1} \dots \sum_{\pi_N} e^{-\beta (\mathcal{E}(1,\pi_1) + \dots + \mathcal{E}(N,\pi_N))}$$

$$\approx \prod_n \sum_{\pi_n} e^{-\beta \mathcal{E}(n,\pi_n)}$$





How can we integrate multiple sources of biological prior knowledge?
MCMC with multiple sources of prior biological knowledge

We model the prior with the Gibbs distribution:

 $P(G|\beta_1,\beta_2) = \frac{e^{-\{E(G_1)\beta_1 + E(G_2)\beta_2\}}}{Z(\beta_1,\beta_2)}$

Where the partition function is:

$$Z(\beta_1, \beta_2) = \sum_{G \in \mathcal{G}} e^{-\{E(G_1)\beta_1 + E(G_2)\beta_2\}}$$



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$$Z(\beta_1,\beta_2) = \sum_{G \in \mathcal{G}} e^{-\{E(G_1)\beta_1 + E(G_2)\beta_2\}}$$

$$E_1(G) = \sum_{n=1}^N \mathcal{E}_1(n, \pi_n[G])$$
$$E_2(G) = \sum_{n=1}^N \mathcal{E}_2(n, \pi_n[G])$$

$$Z = \sum_{G \in \mathcal{G}} e^{-\{\beta_1 E_1(G) + \beta_2 E_2(G)\}}$$

$$\approx \sum_{\pi_1} \dots \sum_{\pi_N} e^{-\{\beta_1 [\mathcal{E}_1(1,\pi_1) + \dots + \mathcal{E}_1(N,\pi_N)] + \beta_2 [\mathcal{E}_2(1,\pi_1) + \dots + \mathcal{E}_2(N,\pi_N)]\}}$$

$$\approx \prod_n \sum_{\pi_n} e^{-\{\beta_1 \mathcal{E}_1(n,\pi_n) + \beta_2 \mathcal{E}_2(n,\pi_n)\}}$$

BioSS

MCMC with multiple sources of prior biological knowledge

Sample graph and the parameters β_1 and β_2

$$A = \min\left\{\frac{P(D|G')P(G'|\beta_1',\beta_2')}{P(D|G)P(G|\beta_1,\beta_2)}, 1\right\}$$

Separate in three samples to improve the acceptance:

- 1. Sample graph with β_1 and β_2 fixed.
- 2. Sample β 1 with graph and β 2 fixed.
- 3. Sample $\beta 2$ with graph and $\beta 1$ fixed.

$$A_{1} = \min\left\{\frac{P(D|G')P(G'|\beta_{1},\beta_{2})}{P(D|G)P(G|\beta_{1},\beta_{2})},1\right\}$$
$$A_{2} = \min\left\{\frac{P(G|\beta'_{1},\beta_{2})}{P(G|\beta_{1},\beta_{2})},1\right\}$$
$$A_{3} = \min\left\{\frac{P(G|\beta_{1},\beta'_{2})}{P(G|\beta_{1},\beta_{2})},1\right\}$$



Bayesian networks with prior biological knowledge

•Prior biological knowledge: Information about the interaction between nodes.

•In our simulations we use two distinct sources of biological prior knowledge.

•Each source of biological prior knowledge is associated with its own trade-off hyperparameter: β_1 and β_2 .

•Trade off hyperparameter indicates how much biological prior information is used.

•Trade off hyperparameters are sampled. They are not set by the user!











I presented the method and how it is supposed to work.

Is it what we get when applying it to real data?















KEGG PATHWAYS are a collection of manually drawn pathway maps representing our knowledge of molecular interactions and reaction networks.



The data and the priors



Define by M_{ij} the total number of times a pair of genes *i* and *j* appears in a pathway, and by m_{ij} the number of times the genes are connected by a (directed) edge in the KEGG pathway. The elements B_{ij} of the prior knowledge matrix are then defined by

$$B_{ij} = \frac{m_{ij}}{M_{ij}} \tag{43}$$

If a pair of genes is not found in any of the KEGG pathways, we set the respective prior association to $B_{ij} = 0.5$, implying that we have no information about this relationship.

















Evaluation 2: TP scores



We set the threshold such that we obtained 5 spurious edges (5 FPs) and counted the corresponding number of true edges (TP count).



We have the data sets and two different sources of prior one of which is random.

How the sampled trade off hyperparameters look like?





And the reconstructed network?





Are the trade off hyperparameters optimal?



Learning the trade off parameters on simulated data

mean and standard deviation of the sampled trade off parameter





- We simulated data from the accepted network strucuture.
- We are sure that we don't have any mismatch between the data and the network we use to calculate the AUC scores.
- Now the sampled trade off parameter is optimal

New evidence for the accepted network

Regulation of Raf-1 by Direct Feedback Phosphorylation. *Molecular Cell*, Vol. 17, 2005 Dougherty et al



Figure 7. A Model for Raf-1 Regulation by Feedback Phosphorylation



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Figure 7. A Model for Raf-1 Regulation by Feedback Phosphorylation



Summary



- Extended method can distinguish between good and bad sources of prior.
- Application to real data leads to significantly improved results.
- Trade off parameters are close to the optimal.
 Differences can be explained by the inconsistencies in the accepted network.

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Reconstructing Gene Regulatory Networks with Bayesian Networks by Combining Expression Data with Multiple Sources of Prior Knowledge

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